ISDA-Online

Thursday, 15 January 2026, 15-17 UTC

“Maths and Methods”

Organizers and Conveners: *Robin Armstrong (Cornell University), Javier Amezcua (ITAM, Mexico; U. of Reading, UK)*

This session is on the fundamental ideas of data assimilation. We invite abstracts dealing with recent advancements in the methodology of data assimilation, as well as the mathematical developments behind these. Traditional methods (variational and Kalman-filter-based) and novel methods (non-parametric filters, particle filters and flows, optimal transport) are encompassed.

Program: (UTC)

15:00 – 15:05	**Opening remarks**
15:05 – 15:25 (17’+3’)	**Efficient continuous nonlinear data assimilation** Peter Jan van Leeuwen
15:25 – 15:45 (17’+3’)	**Inaccuracy of ensemble-based covariance propagation beyond sampling error** Shay Gilpin
15:45 – 16:05 (17’+3’)	**Parametric error forecasts for operational air quality assimilation under highly uncertain emissions** Annika Vogel, Richard Menard, James Abu, Jack Chen
16:05 – 16:25 (17’+3’)	**A novel solution for multivariate updates to state quantities constrained by summation: theory and application to sea ice** Molly M. Wieringa, Jeffrey L. Anderson
16:25 – 16:45 (17’+3’)	**Ensemble Conjugate Transform Inversion (ECTI): A transport-based framework for estimating uncertain model parameters** Hristo G. Chipilski, Derek J. Posselt
16:45 – 16:50	**Closing: Information on upcoming sessions**

Please note:

- When you login to the session before 15:00 UTC, and everything could be quiet, this is most likely because we muted the microphones.
- The times in UTC are approximate. In case of technical problems, we might have to change the order of the presentations.
- **Time Zones**: 15 – 17 UTC

Europe:	04 – 06 pm BST (London)	05 – 07 pm CEST (Berlin)	
Asia/Australia:	11 – 01 am CST (Shanghai)	00 – 02 am JST (Tokyo)	02 – 04 am AEDT (Sydney)
Americas:	08 – 10 am PDT (San Fran.)	09 – 11 am MDT (Denver)	11 – 01 pm EDT (New York)

Efficient continuous nonlinear data assimilation

Peter Jan van Leeuwen[1]
[1]Colorado State University

Filters and smoothers suffer from abrupt updates at observation times or at the beginning of an assimilation window, respectively. Good-old nudging methods do not have this problem, but are ad hoc. However, it is possible to formulate an ensemble data-assimilation method in which the nudging is performed by back-propagating the likelihood via the Kolmogorov backward equation. In this way, the fully nonlinear data assimilation problem is solved exactly. While the backward Kolmogorov equation is too expensive to solve, we can use the so-called small-time approximation introduced in Conti, Van Leeuwen, and Anderson (2025) to provide highly accurate solutions.

This results in a fully nonlinear data-assimilation method that simply propagates an ensemble forward in time, with an extra forcing term for each observation, without separate 'updates'. Examples using simple systems where we know the exact solution, and high-dimensional systems show the performance of this new methodology. Finally, and interestingly, localization is not needed by construction, and correlated observations are easily incorporated.

Inaccuracy of ensemble-based covariance propagation, beyond sampling error

Shay Gilpin[1]
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Modern data assimilation schemes typically use the same discrete dynamical model to evolve the state estimate in time also to approximate the evolution, or propagation, of the estimation error covariance. Ensemble-based methods, such as the ensemble Kalman filter, approximate the evolution of the covariance through the propagation of individual ensemble members. Thus, it is tacitly assumed that if the discrete state propagation and resulting mean state estimates are accurate, then the ensemble-based discrete covariance propagation will be accurate as well, apart from sampling errors due to limited ensemble size. Through a series of numerical experiments supported by analytical results, we demonstrate that this assumption is false when correlation length scales approach grid resolution. We show for states that satisfy advective dynamics, that while the discrete state propagation and ensemble mean state estimates are accurate, the corresponding ensemble covariances can be remarkably inaccurate, well beyond that expected from sampling errors or typical numerical discretization errors. The underlying problem is a fundamental discrepancy between discrete covariance propagation and the continuum covariance dynamics, which we can identify because the exact continuum covariance dynamics are known. Errors in the ensemble covariances, which can be at least one order of magnitude larger than those of the mean state when correlation lengths begin to approach grid scale, cannot be rectified by the usual methods, such as covariance inflation and localization. This work brings to light a fundamental problem for data assimilation schemes that propagate covariances using the same discrete dynamical model used to propagate the state, and thereby opens the door to potential solutions.

Parametric error forecasts for operational air quality assimilation under highly uncertain emissions

Annika Vogel[1,2], Richard Menard[2], James Abu[2], Jack Chen[2]
[1]University of Cologne
[2]Environment and Climate Change Canada

Operational air quality assimilation systems require accurate error estimations representing the complex case-dependent uncertainties during extreme air quality events like wildfires. At the same time, operational forecast systems require high computational efficiency to deliver forecasts to the public. This study explores the potential of a novel assimilation approach, called parametric Kalman filter (PKF), for operational air quality forecasting during extreme air quality events. The PKF offers a computationally efficient alternative to existing ensemble approaches, where the dynamics of the main error parameters are explicitly included in the forecast system. This contribution focuses on the theoretical derivation of error dynamics for near-surface air quality forecasts and its implementation in the operational model GEM-MACH. The theoretical investigation suggests that error standard deviation is a more suitable parameter than error variance for operational models. This is due to simpler forecast equations that have multiple advantages like numerical accuracy, improved process understanding and minor modifications in the forecast model. It is shown that each process in the forecast model has a specific impact on forecast errors which can be investigated using this parametric formulation. The theoretical insights are complemented by results from a wildfire case study in eastern Canada in July 2023 which shows significant impacts on hourly PM2.5 analysis increments compared to the operational setup. Although the current setup of the PKF error forecast remains highly simplified, it enables efficient spreading of sparse observation information in a highly case-dependent and anisotropic way only though improved variance fields.

A novel solution for multivariate updates to state quantities constrained by summation: theory and application to sea ice

Molly M. Wieringa[1], Jeffrey L. Anderson[2]
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Ensemble filtering methods for a modeled system can be implemented in a two-step framework. During the first step, the model ensemble estimate of the observation is updated by direct comparison to an observation; in the second step, the updates to observed variable are regressed onto unobserved state variables using the ensemble error covariance relationship between them. For Gaussian problems in which the unobserved and observed variables are approximately independent, linear regression accurately syncs the model state to changes in the observed space. This work considers a non-Gaussian Earth system application in sea ice models, where the observed variable (e.g. sea ice concentration) is an aggregation of the state variables (e.g. sea ice area fraction in several thickness classes). In this case, the state update is subject to additional constraints not addressed by standard linear regression methods.

A novel solution for updating state variables constrained by their sum is introduced. The solution, based on a normalization of the state, is then applied in a simplified sea ice modeling context and compared to Gaussian and non-Gaussian regression methods and a simple disaggregation approach. The new method produces accurate state updates that more appropriately reflect the observation update calculated for their sum compared to existing regression methods; it is also shown to be more general than disaggregation.

Ensemble Conjugate Transform Inversion (ECTI): A transport-based framework for estimating uncertain model parameters

Hristo G. Chipilski [1], Derek J. Posselt [2]
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[2]NASA Jet Propulsion Laboratory

One of the major challenges in cloud and precipitation modeling is the presence of bounded quantities. For example, precipitation amounts are non-negative and exhibit highly skewed distributions near their lower bounds. Similar constraints apply to cloud microphysical parameters, which govern the evolution of cloud processes in Earth system models. As a result, parameter estimation methods based on linear/Gaussian assumptions struggle to accurately constrain these microphysical parameters from cloud-sensitive observations.

In this talk, we investigate the feasibility of a newly developed framework — the Ensemble Conjugate Transform Inversion (ECTI) — for estimating microphysical parameters within a single-column cloud model. ECTI builds on a recent generalization of Kalman filter theory to non-Gaussian distributions (Chipilski 2025: https://doi.org/10.1175/JAS-D-24-0171.1) and can incorporate learnable transport maps. We will highlight the advantages of ECTI implementations that use normalizing flows, a class of generative AI methods, and compare their performance against the benchmark Ensemble Kalman Inversion (EKI) as well as other nonlinear generalizations.